AIAA Journal
Vol. 45, No. 8, August 2007

Technical Notes

TECHNICAL NOTES are short manuscripts describing new developments or important results of a preliminary nature. These Notes should not exceed 2500 words (where a figure or table counts as 200 words). Following informal review by the Editors, they may be published within a few months of the date of receipt. Style requirements are the same as for regular contributions (see inside back cover).

Wall Effects on Vibration-Induced Particle Motion in a Fluid Cell in Space

Samer Hassan* and Masahiro Kawaji†
University of Toronto, Toronto, Ontario M5S 3E5, Canada

DOI: 10.2514/1.21955

I. Introduction

THERE are many gas–liquid, liquid–liquid, and particle–liquid systems used aboard space platforms such as the International Space Station (ISS) and in future space exploration activities that involve the motion of bubbles, droplets, and particles in microgravity; such systems include material processing devices, life-support systems, thermal management systems, and propulsion systems, among others.

In the absence of gravity, a particle would remain stationary in a liquid even if their densities are drastically different, but the particle can respond to small external vibrations, called g-jitter on space platforms. The vibrations may in turn be used to move the bubbles and particles in a controlled manner and achieve phase separation in a fluid–particle system.

This work addresses the motion of a solid particle in a fluid cell subjected to a sinusoidal vibration with a certain frequency and amplitude. An inviscid model based on the method of images has been used to investigate the vibration-induced motion of a particle oscillating normal to a cell wall in a semi-infinite fluid-filled cell. The model that accounts for the virtual mass force but neglects the viscous drag and history forces is applied to predict the cell wall effect on the particle amplitude.

Coimbra and Rangel [1] derived an analytical model for the periodic motion of a small particle in a viscous fluid. In their study, relative scaling of the virtual mass, Stokes drag, and history forces was presented in terms of the scaling parameter $S = (2\pi f/\nu)(R_o/3)^2$ and the fluid-to-particle density ratio $\alpha = \rho_f/\rho_s$, where f is the vibration frequency, ν is the kinematic viscosity of the fluid, and R_o is the particle radius. They solved the equation of motion analytically and graphically presented the ratio of predicted particle-to-vibration amplitudes in the fixed frame of reference η against S for different density ratios. Their analysis has been verified experimentally by Coimbra et al. [2] and L'Esperance et al. [3], however, the wall effect on the particle amplitude was not addressed.

Recent protein crystal growth experiments conducted by Gamache and Kawaji [4] have shown that small vibrations can induce movements of protein crystals that can in turn cause a significant fluid motion and affect both the growth rates and the final quality of the protein crystals grown in space. There are other observations of protein crystals moving in space experiments [5], possibly due to g-jitter on space platforms.

Although there is a clear need to experimentally investigate the vibration-induced particle motion in a systematic manner, such an experiment is not possible on the ground due to the effect of gravitational sedimentation if there is any density difference between the particle and surrounding fluid. Recently, Hassan et al. [6,7] developed a theoretical model to predict the vibration-induced motion of a spherical particle suspended by a thin wire in a fluid cell under normal gravity, assuming an inviscid fluid and irrotational flowfield. Their inviscid model was able to well predict the amplitudes of different metal particles oscillating in a fluid cell, which were subjected to small horizontal vibrations under normal gravity. Their models also showed a resonance phenomenon in which the particle amplitude would be amplified at a certain cell vibration frequency.

In the present work, an inviscid fluid assumption is made to obtain an analytical expression for the amplitude of a freely moving particle in a semi-infinite fluid cell vibrated in one direction with a certain frequency and amplitude under zero gravity. The inviscid fluid assumption can be justified if the following inequality is valid:

$$f \gg \frac{\nu}{2\pi R_o^2} \tag{1}$$

For example, for a particle of 1-mm diameter immersed in water ($\nu = 10^{-6}\ \mathrm{m^2/s}$), the lower boundary of the cell vibration frequency at which Eq. (1) is valid would be given by $f \gg 0.16$ Hz. The g-jitter encountered aboard space platforms such as the ISS includes vibration frequencies much higher than 0.16 Hz [8]. Thus, the present inviscid analysis of vibration-induced particle motion in fluid systems is expected to apply to many practical situations encountered in space.

II. Theoretical Analysis of the Particle Motion in Zero Gravity

To obtain an expression for the particle amplitude in an inviscid fluid and zero-gravity environment that accounts for the wall proximity effect, it is necessary to analytically solve an equation of particle motion that incorporates the distance H between the particle's center of mass and the nearest cell wall and the particle radius R_o, as shown in Fig. 1. Such an analysis was performed by Hassan et al. [7] using the method of images for a particle suspended by a thin wire from the top of the fluid cell, but their expression for the particle amplitude included a term containing the gravitational acceleration, the length of the wire used to suspend the particle from the top of the fluid cell, and the cell vibration frequency.

The Hassan et al. [7] analysis can be readily modified by omitting the term containing the gravitational acceleration and the length of the wire. This results in the following equation for the particle amplitude A_p in the cell frame of reference:

$$A_p = \frac{(\rho_S - \rho_f)a}{-\left[\rho_S + \rho_f\left\{\frac{G}{K}\left(\frac{1}{R_o^3} + \frac{2}{W^3}\right)\right\}\right]} \tag{2}$$

where a is the cell amplitude, W is the distance between the particle

Received 21 December 2005; revision received 3 March 2007; accepted for publication 6 March 2007.

*Ph.D. Candidate, Department of Chemical Engineering and Applied Chemistry.
†Professor, Department of Chemical Engineering and Applied Chemistry (Corresponding Author).

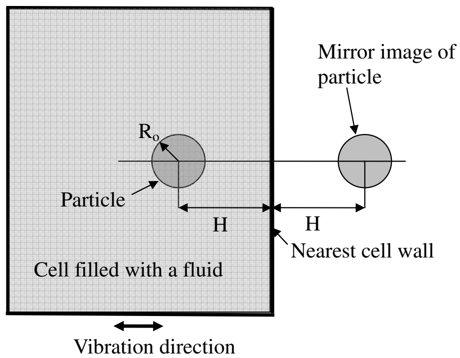

Fig. 1 A solid particle oscillating normal to a cell wall with a mirror image of the particle on the outside.

and its image ($W = 2H$), and G and K are given by the following expressions:

$$G = R_o^3(W - R_o)^3 \tag{3}$$

$$K = 2\left(W^3 - 3W^2 R_o + 3R_o^2 W - 2R_o^3\right) \tag{4}$$

We note that in the limit $H \to \infty$ corresponding to an infinite cell, Eq. (2) reduces to

$$A_{p(H\to\infty)} = \frac{2(\rho_S - \rho_f)a}{-(2\rho_S + \rho_f)} \tag{5}$$

The negative sign in Eq. (2) or Eq. (5) shows that the particle and the cell are moving with an opposite phase for $\rho_S > \rho_f$, whereas they move in phase for $\rho_S < \rho_f$. For both semi-infinite and infinite fluid cells, the inviscid model predicts the particle amplitude A_p to be linearly proportional to the cell vibration amplitude a and independent of the cell vibration frequency.

The wall proximity effect will be studied by examining the ratio of the particle amplitude in a semi-infinite cell to that in an infinite cell, as given by

$$\beta = \frac{A_p}{A_{p(\text{infinite})}} = \frac{(2\rho_S + \rho_f)}{2\left\{\rho_S + \rho_f\left[\frac{G}{K}\left(\frac{1}{R_o^3} + \frac{2}{W^3}\right)\right]\right\}} \tag{6}$$

By introducing the following dimensionless parameters,

$$\tilde{\rho} = \frac{\rho_S}{\rho_f}, \qquad \tilde{H} = \frac{H}{R_o}, \qquad \tilde{A}_p = \frac{A_p}{a} \tag{7}$$

Eqs. (2) and (6) can be written in dimensionless form as follows:

$$\tilde{A}_p = \frac{-(\tilde{\rho} - 1)}{\left[\tilde{\rho} + \frac{1}{2}\left(\frac{\left(1 - \frac{1}{2\tilde{H}}\right)^3}{1 - \frac{3}{2\tilde{H}} + \frac{3}{4\tilde{H}^2} - \frac{1}{4\tilde{H}^3}}\right)\left(1 + \frac{1}{4\tilde{H}^3}\right)\right]} \tag{8}$$

$$\tilde{\beta} = \frac{2\tilde{\rho} + 1}{2\left[\tilde{\rho} + \frac{1}{2}\left(\frac{1 - \frac{1}{2\tilde{H}^3}}{1 - \frac{3}{2\tilde{H}} + \frac{3}{4\tilde{H}^2} - \frac{1}{4\tilde{H}^3}}\right)\left(1 + \frac{1}{4\tilde{H}^3}\right)\right]} \tag{9}$$

The predictions of the preceding expressions will be first compared with the graphical results presented by Coimbra and Rangel [1] for a vibration-induced particle motion in an infinite but viscous liquid. The wall proximity effect on the particle amplitude predicted by Eq. (2) will then be examined in detail.

III. Results and Discussion

Coimbra and Rangel [1] graphically presented the ratio of the predicted particle amplitude to the cell vibration amplitude η in the fixed frame of reference for a large range of the scaling parameter S, varying from 10^{-5} to 10^5, and different fluid-to-particle density ratios. They also predicted that the viscous drag and history forces would be diminished and become less than the virtual mass force for $S > 10$. For density ratios of $\alpha = 0.001$, 0.1, 2, 10, and 1000, the particle amplitudes at $S = 10^5$ predicted by Coimbra and Rangel are 0, 0.14, 1.5, 2.5, and 3.0, whereas those predicted by Eq. (2) in the fixed frame of reference are 0.0015, 0.143, 1.49, 2.498, and 2.97, respectively. For heavy particles, $\alpha = 0.001$ and 0.1, and the particle amplitude was predicted to become constant at S greater than 0.1 and 10, respectively [1]. It is noted that the particle amplitude in the fixed frame of reference is related to that in the cell frame reference given by Eq. (2), as follows:

$$\eta = 1 - A_p/a \tag{10}$$

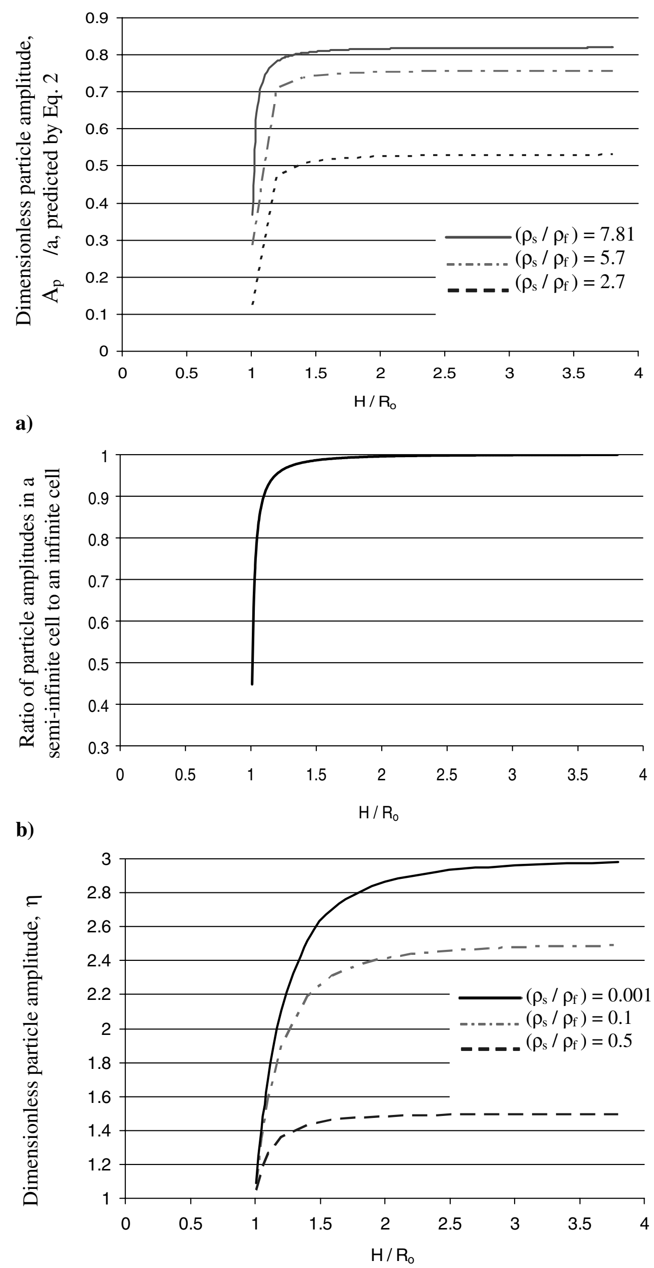

Fig. 2 Wall proximity effects on the variation of a) A_p/a for heavy particles, b) β for a steel particle in water, and c) η for light particles.

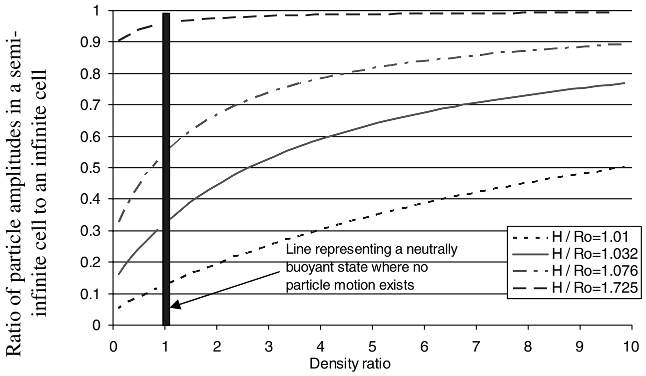

Fig. 3 Variation of the ratio β of the particle amplitude for a semi-infinite cell to that for an infinite cell with the particle-to-fluid density ratio $\tilde{\rho}$ for different values of H/R_o.

The present inviscid model predictions are in good agreement with the predictions of Coimbra and Rangel [1], because the effects of viscous and history forces become much smaller than the virtual mass force at $S = 10^5$. In addition, because the values of η reach asymptotic values even at small to intermediate S values for heavy particles, the present analytical expression for the particle amplitude, Eq. (2), can be applied to many practical situations in space. For example, the S value for a nonviscous fluid (such as water) readily reaches a value greater than 10 if the particle is larger than 2–3 mm in radius and the vibration frequency is higher than about 4 Hz. The vibration frequencies of 4–10 Hz are common high-amplitude components of g-jitter aboard the ISS [8].

The wall proximity effects are shown in Fig. 2. The predictions of Eq. (2) for particles heavier than the fluid ($\tilde{\rho} = 7.81$, 5.7, and 2.7) shown in Fig. 2a indicate that the heavier the particle, the greater the particle amplitude in the cell frame of reference. But in all cases, the particle amplitude would be diminished as the dimensionless distance of the particle to the nearest cell wall, $\tilde{H}$, is reduced.

The wall effect is also illustrated in Fig. 2b, which compares the ratio of the steel particle amplitudes in a water-filled semi-infinite cell with that in an infinite cell ($H/R_o \to \infty$) for any cell vibration frequency. As $\tilde{H}$ is reduced and approaches unity, this amplitude ratio decreases to a value less than 0.5.

The wall effect on the particle amplitude in the fixed frame of reference is shown in Fig. 2c for particles lighter than the fluid ($\tilde{\rho} = 0.1$, 0.5, and 0.001). Again, the particle amplitude in a semi-infinite cell decreases with the decreasing distance between the particle and the nearest cell wall, $\tilde{H}$. The wall proximity effect for particles lighter than the fluid shown in Fig. 2c is shown to be greater than that for the particles heavier than the fluid. One can see clearly from Fig. 2b that the wall effect on the particle amplitude is present only within about two particle radii for a steel particle in water.

Finally, the effect of the particle-to-fluid density ratio at small values of $\tilde{H} < 1.725$ is examined in Fig. 3, in which the ratio of the particle amplitudes β in semi-infinite to infinite cells is plotted against the particle-to-fluid density ratio. A vertical line drawn at $\tilde{\rho} = 1$ represents a neutrally buoyant system in which no relative motion exists between the particle and the cell. When the density ratio is increased, the wall proximity effect is reduced for all values of $\tilde{H}$. For $\tilde{\rho} < 1.0$, the particle amplitude ratio β is shown to decrease more rapidly with $\tilde{H}$, indicating a greater wall proximity effect for lighter particles, which is expected due to a greater effect of hydrodynamic forces on the motion of lighter particles.

References

[1] Coimbra, C. F. M., and Rangel, R. H., "Spherical Particle Motion in Harmonic Stokes Flows," *AIAA Journal*, Vol. 39, No. 9, 2001, pp. 1673–1682.

[2] Coimbra, C. F. M., L'Esperance, D., and Lambert, R. A., "An Experimental Study on Stationary History Effects in High-Frequency Stokes Flows," *Journal of Fluid Mechanics*, Vol. 504, Apr. 2004, pp. 353–363.

[3] L'Esperance, D., Coimbra, C. F. M., and Trolinger, J. D., "Experimental Verification of Fractional History Effects on the Viscous Dynamics of Small Spherical Particles," *Experiments in Fluids*, Vol. 38, No. 1, 2005, pp. 112–116.

[4] Gamache, O., and Kawaji, M., "Experimental Investigation of Marangoni Convection and Vibration-Induced Crystal Motion During Protein Crystal Growth," *Microgravity Science and Technology*, Vol. 16, No. 1, 2004, pp. 342–347.

[5] Chayen, N., Snell, E. H., Helliwell, J. R., and Zagalsky, P. F., "CCD Video Observation of Microgravity Crystallization," *Journal of Crystal Growth*, Vol. 171, No. 1, 1997, pp. 219–225.

[6] Hassan, S., Kawaji, M., Lyubimova, T. P., and Lyubimov, D. V., "Motion of a Sphere Suspended in a Vibrating Liquid-Filled Container," *Journal of Applied Mechanics*, Vol. 73, No. 1, 2006, pp. 72–78.

[7] Hassan, S., Kawaji, M., Lyubimova, T. P., and Lyubimov, D. V., "The Effects of Vibrations on Particle Motion Near a Wall in a Semi-Infinite Fluid Cell," *Journal of Applied Mechanics*, Vol. 73, No. 4, 2006, pp. 610–621.

[8] Jules, K., McPherson, K., Hrovat, K., Kelly, E., and Reckart, T., "A Status Report on the Characterization of the Microgravity Environment of the International Space Station," *Acta Astronautica*, Vol. 55, Nos. 3–9, 2004, pp. 335–364.

S. Aggarwal
Associate Editor